

Constraints on the Cosmological Parameters in the Relativistic Theory of Gravitation

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Abstract

The causality principle imposes the constraints on the cosmological parameters in the relativistic theory of gravitation. As a result, X-matter causes the quite definite cosmological scenario with the alternate acceleration and deceleration and the final recollapse

The relativistic theory of gravitation (RTG) denies the total geometrization and is based on the traditional field approach [1,2]. The gravitation is interpreted as the tensor field generated by the conserved energy-momentum tensor of the matter. There is the background spacetime of the Minkowski's type, which can be restored in any situation. This preserves the unambiguous physical content of the gravitational phenomena and simplifies the unification of the views on the gravitation, on the one hand, and the quantum mechanics, on the other hand.

It is known [2], that the usual cosmological solutions in RTG do not agree with the modern observational data [3,4] because they predict the decelerated character of the cosmological expansion at the present era. The insertion of the cosmological term into field equation destroys the logical structure of the theory because this requires to insert the additional repulsing physical field, which is not affected by the matter.

The generalization of the field equation by the means of the insertion of the scalar component with uniquely defined potential allows the inflationary ex-

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¹ This work was inspired by Prof. A.A. Logunov noted the significance of the causality principle for the cosmological models in RTG. Author appreciate him for the helpful discussion. The work was carried out in the computer algebra system Maple 6 [11]. Author is Lise Meitner Fellow at Technical University of Vienna (Project M611)

panded solutions in RTG [5]. However, there is the purely phenomenological approach resulting in the variety of the cosmological scenarios in the framework of RTG, which is based on the modification of the matter energy-momentum tensor [6]. Such modification is produced by the so-called "dark energy" (X-matter) term with the exotic equation of state $p_x = w_x \rho_x$, where $w_x < 0$ (p_x and ρ_x are the pressure and the density, respectively).

However, Prof. A.A. Logunov kindly suggested [7], that tacking into account the causality principle imposes the constraints on the physically admissible solutions in the latter approach. This allows to choose among the physically meaningful cosmological parameters.

Here we will consider the aspects of the X-matter induced cosmological evolution in RTG, which are inspired by the causality principle. As a result, the defined class of the cosmological scenarios will be selected, and the limits of the maximal scaling factor as well as the approximated value of w_x will be defined.

Let us begin with the usual assumption of homogeneity and isotropy of the effective Riemannian spacetime produced by the action of the gravitational field. The corresponding interval in the spherical coordinates is [2]:

$$ds^2 = d\tau^2 - \alpha a(\tau)^2 \left[dr^2 + r^2 (d\theta^2 + \sin(\theta)^2 d\phi^2) \right], \quad (1)$$

Here τ is the proper time, $a(\tau)$ is the scaling factor; α is the constant of integration. This form of the homogeneous and isotropic interval describing the globally flat spacetime follows from the field equations, which have the form:

$$G_n^m - \frac{m^2}{2} (\delta_n^m + g^{mk} \gamma_{kn} - \frac{1}{2} \delta_n^m g^{pk} \gamma_{pk}) = -8\pi T_n^m, \quad (2)$$

$$D_m \tilde{g}^{mn} = 0, \quad (3)$$

where G_n^m is the Einstein's tensor defined on the effective Riemannian spacetime with the metrics g^{mn} ; γ^{mn} is the metrics of the flat background Minkowski spacetime, D_m is the covariant derivative on the background spacetime, $\tilde{g}^{mn} = \sqrt{-g} g^{mn}$, $c = G = \hbar = 1$, m is the graviton's mass (the inverse transition to the ordinary units corresponds to $m \rightarrow mc^2/\hbar$).

We choose the Galilean metrics as a background. The crucial departure from [6] is tacking into account the causality principle in the framework of RTG [8]: "the causality cone of the effective Riemannian spacetime should be positioned inside the causality cone of the Minkowski spacetime". As a result, the arbitrary isotropic vector u^m obeys:

$$\gamma_{mn}u^m u^n = 0, \quad (4)$$

$$g_{mn}u^m u^n \leq 0, \quad (5)$$

From the Eqs. (1, 4, 5) we have the key condition [2]:

$$a(\tau)^4 - \alpha < 0, \quad (6)$$

which eliminates the cosmological solutions with the eternal expansion and keeps the scenario of IV type in [6]. It is convenient to assign $\alpha = a_{max}^4$, where a_{max} is the maximal value of the scaling factor. Then the cosmological equations are:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi\rho(\tau) - \frac{m^2}{12}\left(2 + \frac{1}{a(\tau)^6} - \frac{3}{a(\tau)^2 a_{max}^4}\right), \quad (7)$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi(3p(\tau) + \rho(\tau)) - \frac{1}{6}m^2\left(1 - \frac{1}{a^6}\right). \quad (8)$$

From the Eq. (7) one can obtain the expression for the minimal density of the matter corresponding to the maximal scaling factor [2]:

$$\rho_{\min} = \frac{m^2}{16\pi}\left(1 - \frac{1}{a_{max}^6}\right). \quad (9)$$

Let's suppose $a_{max} \gg a_0$, where a_0 is the present scaling factor. This assumption is suggested by the accelerated expansion of the universe at the present era.

Then the minimal density is defined by the form of the matter with the slowest decrease produced by the growing scaling factor. As $\rho \propto a(\tau)^{-3(1+w)}$, the X-matter with $w_x < 0$ dominates in the late universe. Hence the Eq. (9) results in:

$$\frac{\Omega_g}{\Omega_x} \cong a_{\max}^{-3\delta}, \quad (10)$$

where $\Omega_g = m^2/(6H_0^2)$ and $\Omega_x = 8\pi\rho_x/(3H_0^2)$ are the density parameters for the gravitons and the X-matter, respectively, H_0 is the Hubble constant; $\delta = 1 + w_x$ is the deviation of the X-matter state parameter from that for the pure cosmological constant.

The Eq. (7) defines the modified cosmic sum rule:

$$\Omega_r + \Omega_m + \Omega_x - \frac{3}{2}\Omega_g \left(1 - \frac{1}{a_{\max}^4}\right) = 1 \quad (11)$$

where $\Omega_m = 8\pi\rho_m/(3H_0^2)$ and $\Omega_r = 8\pi\rho_r/(3H_0^2)$ are the density parameters for the nonrelativistic and relativistic matter with $w=0$ and $1/3$, respectively. One can see, that the sum of the matter densities exceeds the critical density due to the graviton's mass contribution. Although the modern data demonstrates some exceeding $\Omega_{tot} = \Omega_x + \Omega_m + \Omega_r \cong 1.11 \pm 0.07_{-0.12}^{+0.13}$ [9], we suppose that the gravitons induced effect is too small to be revealed in these observations.

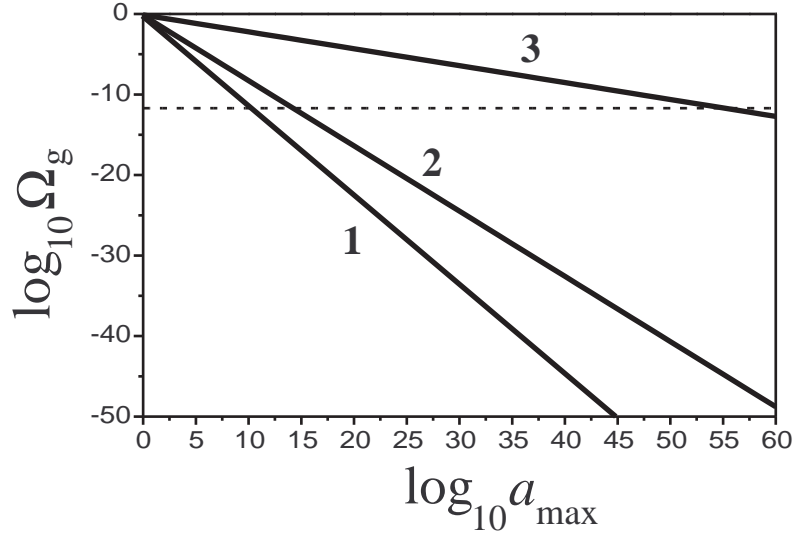
The parameters of the Eq. (10) can be concretized additionally by taking into account the accelerated expansion of the universe at the present era and the observational data from BOOMERANG, MAXIMA and COBE [3,4,9]. The acceleration parameter $q = (d^2a/d\tau^2)|_0/(a_0H_0^2) \cong 0.33 \pm 0.17$, $\Omega_m \cong 0.37 \pm 0.07$, $\Omega_x \cong 0.71 \pm 0.05$.

From the Eqs. (7,8) we have

$$q = \frac{\Omega_x \left(1 - \frac{3}{2}\delta\right) - \frac{1}{2}\Omega_m - \Omega_r}{\Omega_{tot} - \frac{3}{2}\Omega_g}. \quad (12)$$

If the gravitons and the relativistic matter do not contribute in the present state, the combination of observational data and Eq. (12) results in the estimation of δ :

Figure 1. The logarithm of the maximal Ω_g versus the logarithm of the maximal scaling factor for $\delta=0.27$ (1), 0.16 (2), 0.07 (3); $(\Omega_x, \Omega_m)=(0.66,0.44)$, $(0.71,0.37)$, and $(0.76,0.3)$, respectively. The dashed curve is the maximal Ω_g resulted from $a_{min}<a_r$.



$$\delta = \frac{2}{3}(1 - q) - \frac{\Omega_m}{3\Omega_x}(1 + 2q) \cong 0.16^{+0.11}_{-0.09}. \quad (13)$$

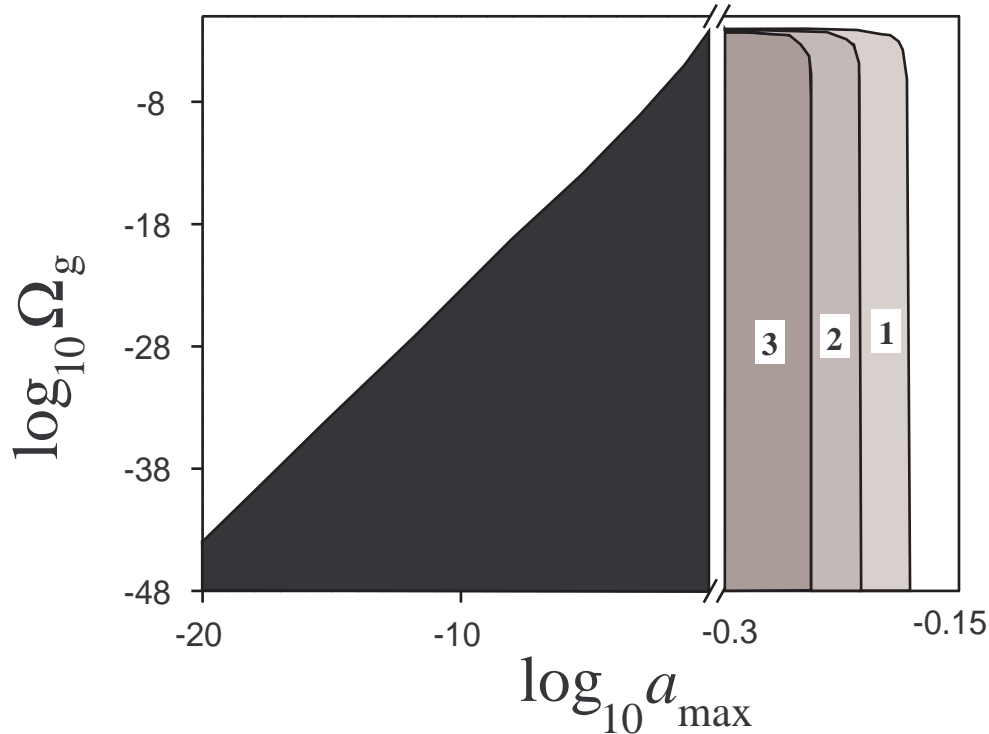
Now we have to estimate the maximal value of Ω_g tacking into account the condition $a_{min}<a_r$, where a_{min} is the minimal scaling factor, a_r is the scaling factor at the end of the radiation dominating era [6]. $a_{min} \approx \sqrt{\Omega_g/2\Omega_r}$ results in $\Omega_g<10^{-11.7}$. This condition in the combination with Eqs. (10, 13) gives Fig. 1. One can see, that $\log_{10}(a_{max})=10\div55$ (with the most probable value in the vicinity of 14) and, it is natural, the approach of w to -1 or Ω_x to 1 increases the maximal scaling factor due to growing negative pressure of the X-matter.

The regions of the accelerated (decelerated) expansion can be found from Eq. (8). The boundaries of these regions are defined by the solutions of the following equation:

$$\Omega_m a^2 + 2\Omega_r a^2 - (2 - \delta)\Omega_x a^{3(2-\delta)} + 2\Omega_g(a^6 - 1) = 0. \quad (14)$$

The corresponding scenario has a complicated loitering character (see [6]): *acceleration* \rightarrow *deceleration* \rightarrow *acceleration* \rightarrow *deceleration* \rightarrow *recollapse*. We live at the era of the second acceleration, which began not long before the present

Figure 2. The regions of the first deceleration for $\delta=0.27$ (1), 0.16 (2), 0.07 (3); $(\Omega_x, \Omega_m)=(0.66,0.44)$, $(0.71,0.37)$, and $(0.76,0.3)$, respectively. The black region is common for all parameters, the end of the deceleration eras is different for the different parameters (filled regions 1, 2 and 3).



time (see Fig. 2). A long first deceleration should have the pronounced observational consequences, for instance, in the large-scale structure formation. The second deceleration era and the recollapse turning point are too remote from us, but the estimated upper limit of the universe age is not too large in the comparison with the so-called "dark era" representing the decay of all known physical processes [10].

In the conclusion, the causality principle in the relativistic theory of gravitation imposes the constraints on the cosmological parameters defining the acceleration behavior of the universe at the present time. The existence of the minimal and maximal scaling factors requires to choose the fixed scenario with complicated loitering behavior. For the observational values of the usual and X- matter densities the deviation of the X-matter state from pure vacuum one is $0.16^{+0.11}_{-0.09}$, which results in the maximal scaling factor $\sim 10^{10} \div 10^{55}$.

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